4 The fund-flow model and the production function

4.1 The concept of production function

The concept of the production function has existed for many years in the field of economic analysis. It started out life as a mutatis mutandis extension to the conceptual corpus related to utility maximisation.[1] At the beginning of the 20th century, the production function started to be used in empirical research with the confluence, on the one hand, of the agronomical tradition for the calculation of response curves and, on the other, the beginnings of the theory of economic optimisation. Over time, the production function concept has been used in many theoretical and mathematical developments, always seeking formulae to aid the producer to make rational decisions in the quest to maximise earnings. The massive estimation of production functions was consolidated in the 1950s with the advances in econometrics and calculating systems. Nonetheless, in the following decade, the model was placed in doubt due to the problems of consistency encountered with the theory of marginal productivity. Despite this, the production function concept has always been widely used and disseminated. It is sufficient to open any manual, or glance through the content of the most economic journals to prove this.

Exclusively for the purposes of this chapter, table 4 classifies the areas of research in which the production function has been employed. This arises from the combination of two criteria: the degree of aggregation of the data used and whether the nature of the research was conceptual or applied.

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[1] The idea of representing technology as a field in the area of merchandise in the same way as consumer preferences, was initially suggested by a certain H. Amstein to Walras in January 1877. Presented as a metaphor, the idea did not prosper. Years later, the idea was to be given due credit with Wicksteed, Barone and, most especially, Johnson. It was the latter who invented the term production function, linking it explicitly to the pure utility theory (Mirowski 1989: 309ff.).

Table 4. Type of production functions

		Scope of the model	
		Macroeconomic	Microeconomic
Object of the research	Theoretical	Analysis of the growth and distribution of income	Criteria of optimum assignation and entrepreneurial rationality
	Applied	Estimation of aggregate production functions: the contribution of the factors and technical change to growth	Assessment of the efficiency of the production process

The definition of a production function, especially in microeconomic research, tends to be accompanied by a long list of assumptions. The most common are the following (Frisch 1965; Ferguson 1969; Johansen 1972):

1. Output is single.
2. The process of transformation is instantaneous: all the inputs intervene at the same initial instant and the complete output emerges immediately.
3. Both the product and the factors may be measured in terms of technical units or, in default, by indices of a quantitative nature.
4. The technique is constant. Consequently the combination of the factors takes place within a given framework of technical knowledge.
5. The production factors specified are continuous and thus there is an infinite number of possible combinations of them.
6. The inputs are totally separable. Or, in other words, there is absolutely no degree of complementarity between them.
7. The functional form is differentiable twice.

The above assumptions is supplemented by the supposition that there is an agent who, given certain preferences, makes decisions to maximize benefits in an environment of perfect competition.

On the basis of the above assumptions, numerous formats for the production function have been proposed. The most generic defines the function in terms of the following expression,

$$Y = F(V_1, V_2, ..., V_n)$$

in which a static framework, V_1, V_2, ..., V_n indicates a certain quantities of factors, Y denotes the amount of product and F the function linking the two magnitudes. This definition will often be juxtaposed with another, establishing the functional relationship with flow rates,

$$y = f(v_1, v_2, ..., v_n).$$

here factors (v_1, v_2, ..., v_n) and the product (y) are amounts per unit of time.[2]

The most widely used mathematical specification of production functions was that proposed by Cobb and Douglas at the end of twenties, although there are indications of an earlier version first established by Johan Gustav Knut Wicksell (1851-1926) in 1895.[3] Nowadays, the Cobb-Douglas version is merely seen as a particular case of the more general CES format.

As is commonly known, Cobb and Douglas (1928) used real data on the US economy between 1899 and 1922, to test the hypothesis of the distribution of income empirically, based on the marginal productivity of factors. Although the econometric adjustments were excellent, the model was soon to be questioned (Mendershausen 1938). Years later, many authors challenged the methodological and theoretical consistency of the Cobb-Douglas aggregate production function, placing its pretensions as a *law* of economics in serious doubt.[4] These objections may equally well be applied to all the variants of this forerunner model.

Representing a production process by way of a production function is a highly simplistic approach: it ignores both the material characteristics of the elements that participate therein and the deployment over time of the activities of the process, as well as the relationship that exists between the process and the environment in which it takes place. Thus the production process is represented as a black box, as shown in figure 46.

Fig. 46. The production process as a black box

A black box approach is commonly found in poorly developed scientific models and theories in which the subject of study is dealt with as if it were a simple unit, ignoring its internal structure. The only interest is in the

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2 The two concepts have been considered equivalent (Frisch 1965). See Georgescu-Roegen (1969, 1972: 280, 1986, 1988, 1990) in which it is demonstrated that this is not the case.

3 See Sandelin (1976). However, there is an earlier version. This can be seen in the work of Johann Heinrich von Thünen (1783-1850), as argued by Heertje (1977).

4 There are solid studies which reveal that the aggregate production function is a mere dynamic expression of the identity of the national accounting system (Fisher 1971; Shaikh 1974, 1980, 1988; Simon 1979; Lavoie 1992: 36). Thus, this function would not account for the stability of the make up of national income over time, but vice versa, the latter would account for the former.

most directly observable behaviour (Bunge 1998). In itself the use of a black box approach is not rejectable as long as it is remembered that an important goal of scientific endeavour is to reduce the degree of opacity of all models.

In the case of the production function, as stated, it is assumed that the period of production is instantaneous. This assumption prevents the observation of the stages of the process: it brings together all of the inputs in the same initial instant, while the output emerges complete at the end. The immediacy between the extremes implies that the process is wanting in structure. From a factual point of view, this approach brings about the risk of formulating a superfluous model. The well tried cookbook recipe analogy could *not* be applied to this model. Although the production function, like a cookbook recipe, is based on a list of ingredients which need to be mixed together, the simile is imperfect in a basic way: all recipes also contain references as to the procedures to follow to prepare the dish, i.e., they indicate the right times to add the different ingredients, together with the cooking times. Moreover, all recipes give the proportions of the different ingredients. Although there may be some room for manoeuvre, at the cook's discretion, it is clear that if the departure from the established proportions is too great, a different dish will be produced. This is something that is ignored by the theory of production. In effect, the isoquants are constructed by grouping together combinations of closer, but also very distant, quantities of factors, yet all supposedly referring to the same output.[5] Although the physical volume of the output may be the same, a significant change in the make up of the diverse ingredients will give rise to different products, that is, with different markets and prices. This is something any manager is very well aware of. Therefore, a doubt is placed on whether all of the points on a given isoquant refer to the same production process. In reality, a specific production process will only be associated with a portion or stretch of points. Thus the isoquants come to be comprised of small lumps of adjacent points (Atkinson and Stiglitz 1969), or also only by isolated points if the process is characterised by strictly fixed coefficients (Koopmans 1951), as shows figure 47.

The figure shows two different processes which combine the same factors to produce a given amount of two different outputs. Points *a* and *b* correspond to the most efficient methods of production for each of these, given that they require the minimum amounts of one or both inputs, with respect to any other process to be found within regions *A* and *B*. Such inef-

[5] This is not merely an abstract stratagem but appears in isoquants constructed on the basis of real data, as is the case in the example of the process of manufacturing hazelnut chocolate given by Frisch (1965).

ficient processes are represented by points which are close to the ends *a* and *b* within the boundaries delimited by the rectilinear segments. In short, any isoquant map may contain empty spaces due to the limited substitution of the factors in order to maintain the homogeneity of the product.

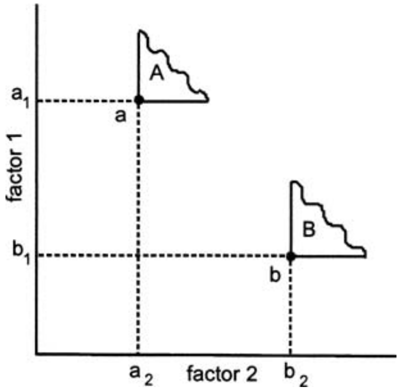

Fig. 47. Isoquants and methods of production

On the other hand, the production function is too conventional. There is a doubtful *ad hoc* proliferation of assumptions employed in its design. Instead of staying within the bounds of the scientific and technological knowledge accumulated in economics and other related disciplines, the model appears to be shielded by a multitude of assumptions which constitute a fictional version of both the nature of the process and the elements involved. Needless to say, such assumptions (continuity, double differentiability, and so on) do have the virtue of guaranteeing results according to the axioms of the rational optimization calculus. All the acceptable mathematical recipes include these suppositions. Indeed, models must show certain returns of scale, patterns of the substitution of factors, and other suitable properties. Thus, making estimates on the basis of real data consists of quantifying the parameters associated to variables, the economic behaviour of which has been prefigured in the mathematical format. In fact, these latter have been chosen for being economically well-behaved.[6] Thus, the specification of the function will depend on the conclusions to be drawn. Conventionalism manifests itself as an artifice and leads to the drawing of priori conclusions.

When dealing with the estimation of microeconomic functions using data from real production processes, the slight differences in the economet-

[6] The reiterated practice of deciding the format of the function by convenience has been classified as a *ceremonial* supposition (Hodgson 1993).

ric adjustments are a mere signal of the way different companies use the same technology. The inputs are combined in very similar proportions per output unit, given that each producer uses their own particular simple variation of the same technology. Nevertheless, these results indicate practically nothing about the specific differences in the process characteristics.

Another important issue with regard to the production function is its supposedly instrumental nature since it is based on data supplied by engineers.[7] In this case, empirical data would describe technically feasible production plans and, if represented by a suitable functional form, will enable the researcher to calculate the economic optimisation values. Indeed, these functions would have the following main uses (Varian 1986: 203):

1. The prediction of the level of production as the amount of the inputs changes.
2. The calculation of the marginal productivity of the different factors specified.

Therefore, the production function would derive from the fortunate encounter of response functions as calculated by the engineers, first of all agrarian ones, and the conventional theory of production which, despite having come into existence as a pure abstraction, would have strong instrumental leanings.[8] For example, the functions of how harvests respond to a dose of fertiliser, formally justified by the continuity of the growth of plants and animals and the divisibility of many of the inputs, ended up being transformed into economic production functions, benefiting from the avalanche of analytical concepts and rules of optimisation revealed by economic theory.

It must be pointed out that, from a historical perspective, agrarian engineering was the first empirical frame for production functions reference. To start with, the relationship between agronomy and microeconomic theory, one has to look back to the work of the German agricultural chemist

[7] For example, Frisch (1965) considers the work on how crops react to different amounts of fertiliser as a precursor to the economic concept of the production function. Another case is the pioneering work by Chenery (1949) on productions functions related to the pipeline transport of gas.

[8] Since the middle of the 19th century, economic theory was progressively concerned with the analysis of maximisation with restrictions. Despite being acceptable for dealing with the very specific issue of the assignation of resources, these new conceptual tools were rapidly applied to all practical economic problems. The result was that the postulate of maximization was elevated, according to its advocates, to the category of *universal law*.

Justus von Liebig.[9] This author enunciated three principles on how crops would react, in terms of production per unit of surface area, to differing amounts of nutrients in the soil (Paris 1992: 1020):

1. The nutritive substances contained in the soil may *not* substitute each other. This postulate is known as the *Law of Minimums*: plant growth is limited by the nutrient of which there is the smallest amount in the soil, even though this latter may contain an excess of the others, in addition to water. Therefore the yield would respond positively to an increased amount of a fertiliser containing this nutrient until another became the limiting factor. This postulate, despite not tackling the issue of the complex interactions among the different nutrients, offers a good macroscopic approach to the relationship between vegetable growth and the chemical richness of the soil.
2. A simple, direct relationship may be established between harvest levels and the amounts of nutrient in the soil or, in other words, between the limiting factor and limited output. Many have interpreted this postulate considering this relationship as lineal. However, on a close reading of von Liebig's work, nothing is found to prevent it from not being so.
3. The response curve reaches a *plateau*, i.e., the yield will no longer respond to increasing doses of fertiliser.

It must be pointed out that his postulates were developed within the context of single crop farming, with the only goal being to establish the maximum amount of output per unit of surface area. Other possible purposes, such as the preservation of the regenerative capacity of the land, are deliberately left aside. It is not a question of disdaining other possible priorities quite simply they are momentarily left out of the horizon of the analysis.

Since the beginning of the 20th century, several different mathematical formulae have been proposed with respect to the principles of fertilising developed by von Liebig. In 1909 the agronomist Mitscherlich presented the following exponential expression:

$$y_i = m\left(1 - ke^{-\beta x_i}\right)$$

in which y_i denotes the production per unit of surface area, x_i the amount of nutrient and m the *plateau* to which the harvest per hectare tends asymp-

9 In 1840, Justus von Liebig (1803-1873) published his influential work *Die organische Chemie in ihrer Anwendung auf Agrikulturchemie und Physiologie*, in which basic aspects of plant nutrition are detailed thus establishing then cornerstone of agricultural chemistry.

totically. In the above equation, β is a constant that expresses the rate of fall of marginal productivity. As can be seen, the model implicitly admits that the remaining nutrients are not affected by limits.

The mathematician Baule extended the model to deal with two or more nutrients. That is,

$$y_i = m(1-k_1e^{-\beta_1 x_{1i}})(1-k_2e^{-\beta_2 x_{2i}}),\ldots,(1-k_ne^{-\beta_n x_{ni}})$$

in which $x_1, x_2, \ldots, x_n$ represent the fertilising substances.

In the mid-nineteen twenties, very simple lineal equations were formulated, such as,

$$y_i = ax_i, \text{ or } y_i = c + ax_i$$

a being a constant of the strict proportionality of the amounts of fertiliser considered and the amount of harvest obtained, and c the corresponding level of production without fertiliser. After the Second World War, such lineal functions became a little more sophisticated, with the proposal of equations of the following kind,

$$y_i = c + \prod_{i=1}^{n} a_i x_i$$

in which y_i is the output per unit of surface area, c the harvest obtained without fertiliser and a_i the coefficients of transformation of the amounts x_i of the fertilisers used together.[10]

More recently, revisiting the von Liebig's work in detail, the following generic formulation of the expression was suggested (Paris, 1992: 1021):

$$y_i = \min\{f_N(N_i, u_{Ni}), f_P(P_i, u_{Pi}), \ldots, f_K(K_i, u_{Ki})\}$$

where, for example, N, P and K are nitrogen, phosphorous and potassium, respectively, and the terms u_{Ni}, u_{Pi} and u_{Ki} refer to the experimental error present. Functions indicated in the equation may or may not be susceptible to lineal specification, and there are diverse assumptions relating to error (additive, multiplicative, or a combination of the two).

[10] A great variety of both technical and economic production functions can be found in Heady and Dillon (1961) and Johnson (1970). It should be stressed that Woodworth (1977) gives a long list of 185 research studies on agricultural production functions carried out in the United States between the 1940s and the 1970s. The number of references from the journal *The American Journal of Agricultural Economics*, previously the *Journal of Farm Economics*, suggests that it is an important vehicle for the dissemination of such experiments.

The above casuistics supplemented by the results of multiple empirical research, has led to the proposal of a hypothetical general curve of how herbaceous crops respond to the application of nutritive substances. This curve is originated by the mere juxtaposition of countless field experiments. See figure 48, in which the top part shows how the crop reacts to larger amount of fertiliser, while the bottom shows the average (continuous line) and marginal product (dotted line) curves.

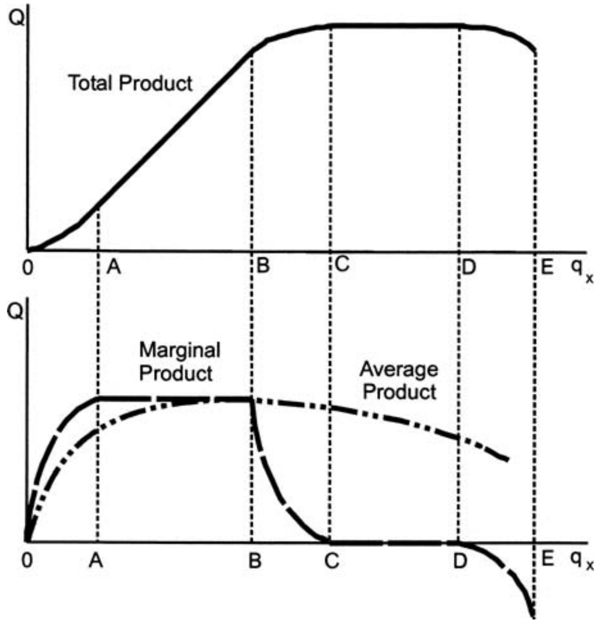

Fig. 48. Fertiliser response curve

Assuming that the fertiliser element is initially present in a limited amount, the response curve could be divided as per the sections shown in the figure. The first three stretches are controversial. Some authors consider a lineal relationship for the whole of the $0C$ pathway satisfactory. Others, to the contrary, opt for a non-lineal relationship: a curve that rises at a decreasing rate. That would imply the assumption that there is a degree of interaction between the different fertilising substances. The figure shows a mixed example: interval $0A$ in which the product grows more than proportionately, followed by a lineal response (section AB) and, lastly, section BC in which the product grows at an ever slower pace.[11]

[11] It must be noted that when looking at plots that have never been fertilised, or have only been so insufficiently, an exponential response curve best fits the

There is a general agreement that the response curves rapidly reach a plateau: the increase in output falls to zero (interval *CD*). The slope of the section leading up to this horizontal interval indicates the efficacy of absorption. The steepness of the slope will lessen if some physical, chemical and/or biological circumstances prevent manure assimilation, meaning that the fertiliser would be wasted. Thus, the specific dose of fertiliser that would lead to such a situation would depend on the type of soil and the farming technique used. To all events, the most relevant aspect of the appearance of this flat interval is the fact that there are maximum doses. Besides that, there is also an absence of any significant degree of substitution amongst nutrients, which can not be confused with interaction between them (Ackello-Ogutu et al. 1985). Finally, section DE represents the unanimous agreement of the agronomists that extremely high doses, unimaginable in any rational fertilising method, end up negatively affecting the yield of the crops.

As can be seen, the lower part of the figure shows the Average and Marginal Product curves. Note that curves are far more complex than the ones proposed by the microeconomists, which try to root production functions in the fieldwork experience of agricultural engineers.

It must be added that the design and estimation of production functions is also a common practice in livestock farming. Such research seeks, on the one hand, to reduce the amount of excess nutrients in the feed rations, the composition of which will vary according to the stage of growth of the animal. On the other hand, the aim is to adjust the outlay on feed to comply with the principles of economic optimisation (Boland et al. 1999).

Unfortunately the confluence of engineering and economic theory does not take into account that the research work by the agronomists:

1. Operates on a different epistemological plane to that of economic analysis.
2. A careful look at the empirical results of agricultural production will not confirm certain economic postulates. Such would be the case, for example, of the substitutability of the factors.

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data. However, one should not loose sight of the fact that the on-going use of nitrogen fertilisers depletes a large part of the bacterial colonies that fix the nitrogen in the soil. Moreover, intensive farming facilitates soil erosion and thus affects the capacity of the land to retain nutrients. Therefore to maintain a high per hectare yield, large amounts of fertiliser have to be applied in a sustained way, even though this may seem excessive. In such a case, the path of the response curve grows at a decreasing rate.

With respect to the first point, it must be said that the theoretical goals of engineering are limited. The response curves attempt to provide an appropriate model, to the extent that they prove useful, with respect to the fragment of reality studied. This pragmatism is characteristic of the technological disciplines. Indeed, the only aim is to detect the existence of a systematic relationship between the variables chosen. After that, such statistical regularities will have to be accounted for with the help of the accumulated knowledge about the underlying physical, chemical or biological processes. This practice is very distant from production functions, which are of a hybrid nature, i.e., both technical and economic (Paris and Knapp 1989). Indeed, after establishing certain general assumptions regarding the process to be studied (such as single output or instantaneous production), and after having selected the factors that will explicitly appear in the function, different mathematical formats are tested, and the one that best supports the verification of the optimisation rules is chosen. The scope of the research is to predict the impact on the output of different variations in the amounts of the inputs considered. The estimated function is then algebraically manipulated, together with any additional information if required, to find its maximum technical limits, the marginal productivities of the factors, etc. and finally the cost functions.

On the other hand, response curves are merely an empirical instrument to calculate the right dose of fertiliser required per unit of surface area, given a plot of land with its own specific edaphologic and climatic features, framed with some agricultural methods of farming. Thereafter, the farmer will merely multiply this unit dose by the appropriate factor according to the size of the plot to be treated. Although comparing of the amounts of fertiliser for the same type of soil and nutrient has shown that there are doses at which the product will increase less than proportionately, or will indeed even fall, the everyday practice of the farmer does not seem to be affected by this. In effect, the aim of research into agricultural engineering is to detect such anomalies and inform the farmer of the dose of fertiliser that will maximise the amount of product obtained per unit of surface area.

The differences between the response curves and production functions are by no means trivial, since they reflect a profound discrepancy in terms of the nature of the research: the former represent a timid first step in a far broader study, while the latter are an authoritative prescription, designed to provide a comprehensive and definitive assessment of the efficiency of a process. Engineering experiences do not tend to *verify* economic postulates but *use* appropriate tools for pragmatic purposes. Therefore, response curves are inclined to be of little formal complexity. In engineering studies, formal simplicity is preferred as, with no more than the empirical evidence, the most complex models are groundless (Bunge 1998).

Moreover it must be remembered that the greatest concern of the agronomist is the design of the field sample. The cost of the experimental programme, the need for it to be representative and the ease of follow-up all require a thorough preliminary study of the sample. Several parcels of the same size are prepared, often adjacent to each other, in order to attempt to determine the right doses of specific material factors (fertiliser, pesticide, water, and so on) by estimating response curves. It must be noted that after a huge number of trials, such functional forms of testing have been refined. Such functions will often merely permit an approximation of the order of magnitude of the appropriate fertiliser. No universal algebraic relationship has been found. They therefore represent no more than a supplementary instrument which, with the help of plant physiology, improves our knowledge of the absorbtion process of fertilisers by plants. This is essential in order to determine the specific dose required. Needless to say that such knowledge may only be applied in other similar agricultural and climatic contexts.

Prior to closing this brief review of the methodological and theoretical problems of the production function, the postulate of the separability of inputs may be examined. Whether it is in agricultural or industrial processes, the factors tend to present a very high degree of complementarity. Here the notion of the *limitational* factor should be restored: the input whose increase is a necessary, though not in itself sufficient, condition for the rise of the amount of product (Georgescu-Roegen 1966). In other words, its impact on the level of production will depend on the increase of the amounts used of at least one of the other factors. This acknowledges the complementarity that characterises all production processes: tasks are an inextricably combination of specific machines and work, along with certain flows. For each task, the technical ratio of the worker and the system of machinery that he/she operates or supervises is only satisfactory within certain very narrow limits. To alter the worker/machine ratio, fixing one of the factors and adjusting the other, would not help increase the output and could, indeed, be absurd from an operational point of view.[12] The difficulty of separating factors means that their respective marginal productivities could not be defined (Pasinetti 1980).

Another problem is related to the presumed law of diminishing returns of the factors. Using other words, this postulate was established by Ricardo

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[12] Care should be taken not to confusing complementarity and functional interaction. While the former refers to the permanent cooperation of production inputs, the latter speaks of the reciprocal implications of the different environments involved in the management of a production process (or of a firm: marketing, finance and so on).

with regard to the land rent question (or the income distribution). Observing what happened in farming in his time, Ricardo believed that the growth of the productive capacity of the agrarian sector would progressively fall further and further: sooner or later, the output from agriculture would shrink below the needs of the growing (urban) population and the requirements of industry.[13]

The modern version of this presumed law should not be confused with the hypothesis of diminishing returns of *factors* as advocated by Ricardo. Indeed, in accordance with him, this postulate is acceptable if, and only if, the volume of output changes when the amount of a given factor varies, provided that quantities of the other factors remain invariant and yield the same productive services. This situation is associated with activities involving a non-reproducible invariant factor, such as agricultural land, which presents a different productive response according to the intensity of use (in a short term horizon). Moreover, the hypothesis enunciated by Ricardo may also be understood as decreasing returns of *scale*: the different productive quality (fertility) of land yields to decreasing returns of scale (or cultivated surface). In both cases, the absence of technical change and the non-reproducibility of the factor (land) are strict conditions for the appearance of diminishing returns.

It should be remembered that Ricardo did not use terms such as decreasing returns of factors or scale. These concepts did not exist in his time. Ricardo's analysis of decreasing returns is the outcome of the economic process of ordering fertility, as well as the declining response of land to the growing intensity of cropping (intensive and extensive margin). It should be added that weather and relative prices may modify the crops sown and, therefore, the fertility arrangement and factor returns could change as well.

As stated, Ricardo's hypothesis is critically based on the implicit assumption that there are no technical changes. Regrettably, Ricardo did not witness the revolution in agricultural productivity given that it started in the middle of the 19th century. Afterwards, it has been difficult to sustain the validity of the presumed law of decreasing returns for a lack of sufficient empirical support.

It should be noted that flexible behaviour is a typical feature of living beings (soil, plants, and animals). Moreover, the intensity of use especially affects the performance of non-reproducible goods (they can show congestion problems, for example). In contrast, machines are reproducible goods and have a very lineal behaviour over a very broad range of use intensity. From that, leaving technical changes aside, the complementary relation-

[13] This trend is only workable for the primary sector as a whole, not for a specific product (given that certain crops may be extended at the expense of others).

ship and lineal responses in production are predominant in the short-term approach.

Criticism about the theory of diminishing returns does not mean advocating constant returns as the most common situation in reality. All the discussion in preceding chapters highlights the importance of the increasing performance of the line processes. Firstly, Adam Smith made it clear that the technical division of labour would increase the skill of the worker and so reduce the execution time of production operations and thus also the idle periods between tasks. This improvement of the productivity of manual labour does not, in principle, require any great change to the tools used. Although the amount of inflow will grow in proportion to the increase in output produced, the specialisation of the worker reduces the man-hour requirement per output unit as the flow expands. To all events, the increasing technical division of work also stimulates improvements in the design of the tools and technological innovation (the invention of more efficient specific machinery). Secondly, Charles Babbage and John Rae insisted on the ability of the Factory System to reduce the idle times of the funds, a fundamental goal given the large number of fixed funds present in industrial processes. Marx also stressed the importance of the so-called increasing returns that could be achieved by combining technical improvements to the equipment (faster operating speed or greater size) with improvements to the organization of work (training, coordination of workers, etc.). It must be remembered that for the Classical school economists the price of merchandise is determined by the conditions of production and not by any assumed functional link between the returns and the amount of output generated.

Thirdly, Sraffa (1926, 1986) distinguished between increasing returns with a constant factor and increasing returns of scale. The former only appear when the input which is constant (i.e., its amount cannot be changed at will) is *indivisible*. In that situation, an increase in the quantities of the variable inputs used means achieving increasingly more efficient proportions with respect to the indivisible input. From that output will grow until attaining an optimum balance of the two types of inputs. However, the constant factor may be divisible, as is the case with land, giving rise to the increasing returns. Indeed, the amount of land required may be adjusted according to the availability of the variable factors.

Increasing returns of scale are associated with an increase in all inputs. They can only occur when no constant factor is present. With divisible inputs, increasing output will always be related to changes in the scale of production. If such increased production benefits just one single firm, it will come to hold a dominant position in the market.

Returning to the rationale that supports the presumed law of diminishing returns, reference tends to be made to the of diseconomies of scale that arise as firm increases in size. An in-depth look at this question shows that the limiting factor is the *rate* of growth and not the absolute firm size. In effect, it becomes progressively more and more difficult to sustain a high rate of expansion. The gradual exhausting of the more profitable investment projects reduces the rate of returns for the company and, at the same time, its growth.

Finally, before closing this chapter, it should be recalled that the production function model groups the elements that participate in the process in a few qualitative factors (often capital and labour). As is well known, such aggregation presents serious problems, especially in the case of capital:

> The goods (or services of long-lived goods) can be expressed as quantities of labour time performed at various dates in the past when the goods were produced. But we can no more sum this labour time in terms of man-hours than we can sum apples or bilberries in terms of fruit. To sum the cost of expenditures of labour time at various dates it is necessary to accumulate them from the time when they were performed till to-day at some rate of interest, subtracting from this cost the yield with which they must be credited for value that they have produced meanwhile (Robinson 1955: 68).

This aggregation of goods cannot be measured in physical terms, that is, without taking into account the rate of return or, in other words, the changes in the income distribution. Moreover, on occasion it is even hard to understand certain specific results. For example,

> the interest in calculating the elasticity of production with respect to the capacity of management is notably limited by the fact that it is a variable that cannot be measured directly and it is very hard to imagine what an x% increase therein would truly represent (Faudry 1974: 712-713).

If the partial equilibrium analysis considers each capital item separately and account for it, this will provide a truer representation of reality, although it must be said that it would be hard to meet with the requirements of the well-behaved economic models.

4.2 The process in line and the production function

The dissatisfaction caused by the methodological and theoretical weaknesses of the production function justifies developing an alternative approach. Circumscribed by partial equilibrium, with its pros and cons, the funds and flows model allows a better approximation to production processes following the generic concept of the production function. As has